

THE CHINESE UNIVERSITY OF HONG KONG  
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018

Tutorial Classwork 8

1. Let  $X$  be a  $T_1$  connected space. Show that  $X$  is either a singleton or an infinite set.
2. Let  $p : X \rightarrow Y$  be a quotient map. Suppose that  $p^{-1}(\{y\})$  is connected for any  $y \in Y$ . Show that  $X$  is also connected.
3. (a) Let  $A$  and  $B$  be proper subsets of the topological spaces  $X$  and  $Y$  respectively. Show that if  $X$  and  $Y$  are connected, then  $(X \times Y) \setminus (A \times B)$  is also connected.  
(b) \* Hence, show that  $\mathbb{R}^n$  is not homeomorphic to  $\mathbb{R}$  for any  $n > 1$ .  
(Hint:  $\mathbb{R} \setminus \{0\}$  is disconnected.)